

Holographic Correlators for General Gauge Mediation

Riccardo Argurio^{1,2} Matteo Bertolini^{3,4}, Lorenzo Di Pietro³,
Flavio Porri³ and Diego Redigolo^{1,2}

¹Physique Théorique et Mathématique
Université Libre de Bruxelles, C.P. 231, 1050 Bruxelles, Belgium

²International Solvay Institutes, Brussels, Belgium

³SISSA and INFN - Sezione di Trieste
Via Bonomea 265; I 34136 Trieste, Italy

⁴International Centre for Theoretical Physics (ICTP)
Strada Costiera 11; I 34014 Trieste, Italy

Abstract

We use holographic techniques to compute two-point functions of operators belonging to a conserved current supermultiplet in theories which break supersymmetry at strong coupling. These are the relevant quantities one has to compute in models of gauge mediation to determine the soft spectrum in supersymmetric extensions of the Standard Model (SSM). Such holographic approach can be used for diverse gravitational backgrounds, but here we focus, for definiteness, on asymptotically AdS backgrounds. After presenting the general framework, we apply our formulas to two explicit examples which differ by the nature of the SSM gauginos, which have Dirac or Majorana masses, corresponding to models that respectively preserve or break R-symmetry.

1 Introduction and motivation

The AdS/CFT duality [1, 2, 3] has proven to be an incredibly powerful tool to study properties of strongly coupled gauge theories in terms of dual weakly coupled gravitational backgrounds. The primary objects the basic dictionary allows one to compute are correlators of gauge invariant operators.

Our aim in this work is to apply holographic techniques (i.e. holographic renormalization [4, 5, 6]) to compute two-point functions of gauge invariant operators related by SUSY transformations in a strongly coupled theory, and see how these correlators differ when SUSY is broken at low energies, i.e. either spontaneously or softly. The leading application we have in mind is General Gauge Mediation [7] (GGM), where two-point functions of operators belonging to a conserved current supermultiplet encode all the information that is needed to derive the soft spectrum of superpartners.

In this work, we will focus our attention on asymptotically AdS (AAdS) spacetimes, which correspond to dual theories with a non-trivial superconformal fixed point in the UV.¹ The operators in the current supermultiplet are dual to bulk fields belonging to a vector multiplet of a $\mathcal{N} = 2$ gauged supergravity in 5 dimensions. Our task will be to compute two-point correlators from the linear fluctuations of such fields over a background that breaks SUSY.

An interesting class of backgrounds can be obtained by considering an effective gauged supergravity Lagrangian describing the interaction of the $\mathcal{N} = 2$ gravity multiplet with one hypermultiplet. The latter, known as the universal hypermultiplet, contains the dilaton, and another scalar related to a squashing mode, which has $m^2 = -3$ (in AdS units) and is charged under the R-symmetry. We will show that turning on the charged scalar is a necessary condition in order to get, eventually, Majorana masses for the Supersymmetric Standard Model (SSM) gauginos. On the contrary, pure dilatonic backgrounds, which as such preserve the R-symmetry, can (and do, in general) provide a Dirac mass for gauginos. The rest of the paper is organized as follows.

In section 2 we first briefly review the GGM formalism. Then, we discuss the holographic recipe to compute GGM correlators by means of the renormalized action for a 5d massless vector multiplet, the bulk multiplet dual to the current superfield. In doing so,

¹Strictly speaking, theories which are superconformal cannot break SUSY spontaneously, because $\langle T_{\mu\nu} \rangle = \epsilon \eta_{\mu\nu}$ contradicts $T_{\mu}^{\mu} = 0$. The set ups we consider will be thought of as toy models where the superconformal symmetry is broken explicitly, though often in a strongly coupled way.

we explain in detail how to use holographic renormalization for such fields (for previous works using a similar philosophy see [8, 9]).

In section 3 we present a 5d $\mathcal{N} = 2$ gauged supergravity model which is the simplest possible model containing all necessary ingredients to let us treat several qualitatively different examples, all arising as consistent solutions of the same 5d equations of motion.

As a warm-up, we start in section 4 by considering a pure (and hence supersymmetric) AdS background. The holographic GGM functions are those of a supersymmetric field theory. We use the holographic computation to check the validity of our approach, and as useful reference for the more interesting examples we consider afterwards.

In section 5 we turn to a dilaton-domain wall background, originally found in [10, 11] (see also [12]) in the context of full fledged type IIB supergravity, used at that time as a candidate gravitational dual for confining theories. The background breaks all supersymmetry but preserves the R-symmetry. We find non-vanishing values for the sfermion masses while, consistently, SSM gauginos do not acquire a (Majorana) mass term. Interestingly, we find a pole at zero momentum for one fermionic correlator, which signals the presence of a Dirac-like (hence R-symmetry preserving) mass for the gauginos [13, 14]. Notice that in our strongly coupled theory, such Dirac mass contribution arises as a consequence of strong dynamics in the hidden sector.

Finally, in section 6, we turn-on a small R-symmetry breaking scalar profile, that we treat linearly and without backreaction on the dilaton-domain wall background. The linear approximation is sufficient to show how things change significantly. Indeed, we show that in this case a Majorana mass for the gauginos is generated. Moreover, the pole at zero momentum of the fermionic correlator, responsible for a Dirac-like contribution to gaugino masses in the pure dilatonic background, automatically disappears, in remarkable agreement with field theory expectations [13, 14].

We conclude in section 7 with a summary of our results and an outlook on possible further applications.

2 Holographic renormalization of a vector multiplet

In general gauge mediation [7] the basic objects one has to compute are correlators of operators belonging to the hidden sector current supermultiplet. The 5d gravity dual to such current superfield is a $\mathcal{N} = 2$ massless vector multiplet. In this section we review

why these claims are correct, write down the correlators we will be interested in, and then provide the holographic recipe to compute them.

2.1 Correlators for General Gauge Mediation

Any gauge mediation model can be visualized as a supersymmetric Standard Model visible sector whose gauge degrees of freedom couple to the hidden sector as

$$\int d^4x d^4\theta g V \mathcal{J}, \quad (1)$$

at leading order in the SSM gauge coupling constant g . In the above formula V is a SSM vector multiplet (here and below we will suppress indices and effectively assume a $U(1)$ SSM gauge group, for simplicity), and \mathcal{J} is a $\mathcal{N} = 1$ multiplet of a conserved current.

In 4d, such a multiplet contains a scalar operator J of conformal dimension $\Delta_0 = 2$, a fermionic operator j_α of conformal dimension $\Delta_{1/2} = 5/2$ and a vector operator j_i of conformal dimension $\Delta_1 = 3$.

The current multiplet is associated to a global symmetry of the hidden sector that one has then to weakly gauge and identify with the visible sector gauge group. The soft spectrum at low energies is completely determined by two-point functions between the hidden sector currents, which in Euclidean momentum-space read

$$\langle J(k) J(-k) \rangle = C_0(k^2), \quad (2)$$

$$\langle j_\alpha(k) \bar{j}_{\dot{\alpha}}(-k) \rangle = -\sigma_{\alpha\dot{\alpha}}^i k_i C_{1/2}(k^2), \quad (3)$$

$$\langle j_i(k) j_j(-k) \rangle = (k_i k_j - \eta_{ij} k^2) C_1(k^2), \quad (4)$$

$$\langle j_\alpha(k) j_\beta(-k) \rangle = \epsilon_{\alpha\beta} B_{1/2}(k^2), \quad (5)$$

where a factor of $(2\pi)^4 \delta^{(4)}(0)$ is understood. The C_s functions are real and dimensionless while $B_{1/2}$ is in general complex, breaks R-symmetry, and has the dimension of a mass. Unbroken supersymmetry dictates

$$C_0(k^2) = C_{1/2}(k^2) = C_1(k^2), \quad B_{1/2}(k^2) = 0. \quad (6)$$

Expanding the effective Lagrangian of the gauge supermultiplet in the gauge coupling g , one easily sees that the soft masses for sfermions and gauginos can be expressed in terms

4d op.	Δ	5d field	scaling	AdS mass
$J(x)$	$\Delta_0 = 2$	$D(z, x)$	$D(z, x) \simeq z^2 \ln z d_0(x)$	$m_D^2 = \Delta_0(\Delta_0 - 4) = -4$
$j_\alpha(x)$	$\Delta_{1/2} = 5/2$	$\lambda(z, x)$	$\lambda(z, x) \simeq z^{3/2} \lambda_0(x)$	$ m_\lambda = \Delta_{1/2} - 2 = 1/2$
$j_i(x)$	$\Delta_1 = 3$	$A_\mu(z, x)$	$A_\mu(z, x) \simeq a_{0\mu}(x)$	$m_A = (\Delta_1 - 2)^2 - 1 = 0$

Table 1: $4d \mathcal{N} = 1$ current multiplet and dual supergravity fields

of above correlators as

$$m_f^2 = -g^4 \int \frac{d^4 k}{(2\pi)^4} \frac{1}{k^2} (3C_1(k^2) - 4C_{1/2}(k^2) + C_0(k^2)) , \quad (7)$$

$$m_\lambda = g^2 B_{1/2}(0) . \quad (8)$$

In general, one might like to consider hidden sectors which break supersymmetry at strong coupling, and this makes the problem of computing the correlators of the hidden sector currents a difficult one from the field theoretical point of view.

Our purpose is then to provide the correct recipe to compute GGM correlators (2)-(5) using the AdS/CFT correspondence. In what follows we discuss the various steps one should pursue to reach this goal.

First, one should consider a $5d$ supergravity background being possibly a stable solution of (a consistent truncation of) $10d$ Type IIB supergravity compactified on some $5d$ internal manifold. This background represents, holographically, our strongly coupled hidden sector. Since this background is the source of supersymmetry breaking, we will focus on solutions which do not preserve any supersymmetry. For the time being, we will not need to specify any detail of the background, except that we take it to be asymptotically AdS, that is

$$ds^2 \underset{z \rightarrow 0}{\simeq} \frac{1}{z^2} (dz^2 + (dx^i)^2) . \quad (9)$$

On such background we should then study the fluctuations for the fields dual to the conserved $U(1)$ current multiplet \mathcal{J} . According to the AdS/CFT field/operator correspondence, the $5d$ supergravity multiplet dual to a current supermultiplet \mathcal{J} is a $\mathcal{N} = 2$ massless vector multiplet (D, λ, A_μ) , as detailed in Table 1 (for asymptotically AdS backgrounds, the scaling at the boundary is directly related to the mass of supergravity fields).

On general grounds the existence of such a multiplet is ensured as long as the internal manifold has some isometries. In the context of the full $10d$ theory it would be a difficult

task to identify the correct $10d$ fluctuations corresponding to the $5d$ vector multiplet [15].² For this reason we will consider consistent truncations of the full theory to $5d$ gauged supergravity. Notice, though, that since we want to compute two-point functions of the dual operators, we need to consider the action for the vector multiplet only to quadratic order in the fields (but of course to all order in the background fields).

Given a $5d$ supergravity action, we have to apply the holographic renormalization procedure [4, 5, 6] in order to obtain a finite well defined on-shell boundary action. At the end of this procedure we will get a renormalized action $S_{\text{ren}}[d_0, \lambda_0, a_0]$ quadratic in the sources. The two point functions are then computed deriving twice with respect to the sources.

We recall that the leading boundary mode for the scalar, d_0 , can be directly identified with the source for the operator J . Similarly for the vector, with the gauge choice $A_z = 0$, the leading boundary mode a_{0i} is the source for the $4d$ conserved vector current at the boundary. As for the spinor, it can always be written in terms of $4d$ Weyl spinors of opposite chirality [18]

$$\lambda = \begin{pmatrix} \chi \\ \bar{\xi} \end{pmatrix}. \quad (10)$$

Choosing the sign of the mass $|m_\lambda| = 1/2$ we are fixing the chirality of the leading mode in the near boundary expansion. As we will review, choosing $m_\lambda = -1/2$ the leading mode at the boundary has negative chirality $\lambda \simeq z^{3/2}\bar{\xi}_0$. Using the holographic dictionary, the GGM functions can then be determined from the renormalized boundary action as

$$C_0(k^2) = -\frac{\delta^2 S_{\text{ren}}}{\delta d_0 \delta d_0}, \quad C_{1/2}(k^2) = -\frac{\bar{\sigma}_i^{\dot{\alpha}\alpha} k^i}{2k^2} \frac{\delta^2 S_{\text{ren}}}{\delta \xi_0^\alpha \delta \bar{\xi}_0^{\dot{\alpha}}}, \quad C_1(k^2) = \frac{\eta_{ij}}{3k^2} \frac{\delta^2 S_{\text{ren}}}{\delta a_{0i} \delta a_{j0}}, \quad (11)$$

$$B_{1/2}(k^2) = -\frac{\epsilon^{\alpha\beta}}{2} \frac{\delta^2 S_{\text{ren}}}{\delta \xi_0^\alpha \delta \xi_0^\beta}. \quad (12)$$

In the remainder of this section, we will derive the results above.

2.2 Two-point functions from the renormalized boundary action

In what follows we want to show in some detail how, given a certain background, one can use the procedure of holographic renormalization to calculate two-point functions of

²At least for the standard case of $\text{AdS}_5 \times S^5$ the problem has been fully solved in [16, 17].

current operators (11)-(12). The starting point is given by the interactions of the vector multiplet with the background, encoded in the part of the Lagrangian which is quadratic in the vector multiplet.

Near the boundary of the background geometry we assume that all scalars that might have a non-trivial profile vanish sufficiently fast, so that all their interactions with the vector multiplet can be neglected in this limit. Therefore, near the boundary the (Euclidean) quadratic Lagrangian approaches that of a vector multiplet minimally coupled to the background

$$L_{\text{quad}} \underset{z \rightarrow 0}{\simeq} L_{\text{min}} = \frac{1}{2}(G^{\mu\nu} \partial_\mu D \partial_\nu D - 4D^2) + \frac{1}{4}F_{\mu\nu}F^{\mu\nu} + \frac{1}{2}(\bar{\lambda} \not{D} \lambda + c.c.) - \frac{1}{2}\bar{\lambda}\lambda . \quad (13)$$

Consequently the equations near the boundary approach the AdS form

$$(\square_{AdS} + 4)D \underset{z \rightarrow 0}{\simeq} 0 , \quad (14)$$

$$(\text{Max})_{AdS} A_i \underset{z \rightarrow 0}{\simeq} 0 , \quad (15)$$

$$(\not{D}_{AdS} - \frac{1}{2})\lambda \underset{z \rightarrow 0}{\simeq} 0 . \quad (16)$$

where we fixed the 5d Coulomb gauge in which $A_z = 0$ and the 4d Lorentz gauge $\partial^i A_i = 0^3$, and the differential operators above take the form

$$\square_{AdS} = z^2 \partial_z^2 - 3z \partial_z + z^2 (\partial_i)^2 , \quad (17)$$

$$(\text{Max})_{AdS} = z^2 \partial_z^2 - z \partial_z + z^2 (\partial_i)^2 , \quad (18)$$

$$\not{D}_{AdS} = z \gamma^z \partial_z - 2\gamma_z + z \gamma^i \partial_i . \quad (19)$$

The spinor equation can be rewritten in terms of Weyl components as

$$\begin{cases} z \partial_z \chi + i z \sigma^i \partial_i \bar{\xi} - \frac{5}{2} \chi \underset{z \rightarrow 0}{\simeq} 0 \\ -z \partial_z \bar{\xi} + i z \bar{\sigma}^i \partial_i \chi + \frac{3}{2} \bar{\xi} \underset{z \rightarrow 0}{\simeq} 0 . \end{cases} \quad (20)$$

Correspondingly, the asymptotic behavior of the supergravity fields near the boundary

³Notice that with this gauge choice we can compute only the coefficient of η_{ij} in the vector current correlator; the additional term can obviously be inferred by current conservation.

takes the following form⁴

$$D(z, k) \underset{z \rightarrow 0}{\simeq} z^2 \left(d_0(k) \ln(z\Lambda) + \tilde{d}_0 + O(z^2) \right) , \quad (21)$$

$$A_i(z, k) \underset{z \rightarrow 0}{\simeq} a_{i0}(k) + z^2(\tilde{a}_{i2}(k) + a_{i2}(k) \ln(z\Lambda)) + O(z^4) , \quad (22)$$

$$\begin{cases} \bar{\xi}(z, k) \underset{z \rightarrow 0}{\simeq} z^{3/2} \left(\bar{\xi}_0(k) + z^2(\bar{\xi}_2(k) + \bar{\xi}_2(k) \ln(z\Lambda)) + O(z^4) \right) , \\ \chi(z, k) \underset{z \rightarrow 0}{\simeq} z^{5/2} (\tilde{\chi}_1(k) + \chi_1(k) \ln(z\Lambda) + O(z^2)) . \end{cases} \quad (23)$$

Note that in the scalar case the leading term at the boundary has a logarithmic scaling. This is a peculiar feature related to the fact that $m_D^2 = -4$ saturates the stability bound for a scalar field in AdS₅ [19, 20]. In the fermionic case the choice $m_\lambda = -1/2$ implies that the leading mode at the boundary has negative chirality, as anticipated.

Every coefficient in the expansion is a function of the momentum k , and the variational principle of the supergravity theory is defined by fixing the leading modes at the boundary $(d_0, \bar{\xi}_0, a_{i0})$ and letting all other coefficients free to vary independently. Substituting the ansatz in the asymptotic equations of motion (17)-(19) one can determine the on-shell values of all coefficients but tilded ones (i.e. the source terms), as local (i.e. polynomial in k) functions of the leading modes $(d_0, \bar{\xi}_0, a_{i0})$. In particular we find

$$a_{i2} = \frac{k^2}{2} a_{i0}(k) , \quad \bar{\xi}_2 = \frac{k^2}{2} \bar{\xi}_0(k) , \quad \chi_1 = -\sigma^i k_i \bar{\xi}_0(k) . \quad (24)$$

Conversely, the subleading modes in the near boundary expansion are not determined by the near boundary analysis and in general will be non-local functions of the external momenta which can be derived from the exact solutions of the equations of motion.

When evaluated on-shell, the supergravity action (13) reduces to the boundary terms, which are in general divergent in the limit $z \rightarrow 0$, and have to be regularized. This can be done considering a cutoff surface $z = \epsilon$, for which, after Fourier transformation on the $4d$ coordinates, the boundary terms become

$$S_{\text{reg}} = - \int_{z=\epsilon} \frac{d^4 k}{(2\pi)^4} \frac{1}{2} \left[\epsilon^{-4} (Dz \partial_z D)_{z=\epsilon} + \epsilon^{-2} (A_i z \partial_z A^i)_{z=\epsilon} - \epsilon^{-4} (\xi \chi + \bar{\chi} \bar{\xi})_{z=\epsilon} \right] . \quad (25)$$

Note that the fermionic boundary term [21, 22] is reminiscent of a Dirac mass term.

⁴In order to define the logarithmic mode in the bulk we have to introduce an energy scale Λ which gets identified with the RG scale of the boundary theory.

Plugging the near boundary expansion and taking into account the on-shell relations between the various coefficients, we can collect both a divergent and a finite contribution in the regularized action (25)

$$S_{\text{reg}}|_{\text{div}} = - \int_{z=\epsilon} \frac{d^4 k}{(2\pi)^4} \frac{1}{2} \ln(\epsilon\Lambda) [2 \ln(\epsilon\Lambda) d_0^2 + 4d_0 \tilde{d}_0 + d_0^2 + a_0^i k^2 a_{0i} + 2\xi_0 \sigma^i k_i \bar{\xi}_0] ,$$

$$S_{\text{reg}}|_{\text{finite}} = - \int \frac{d^4 k}{(2\pi)^4} \frac{1}{2} [2\tilde{d}_0^2 + d_0 \tilde{d}_0 + 2a_0^i \tilde{a}_{i2} + a_0^i \frac{k^2}{2} a_{i0} - (\xi_0 \tilde{\chi}_1 + \bar{\chi}_1 \bar{\xi}_0)] .$$

The divergent terms can be subtracted by means of the following covariant counterterms

$$S_{\text{ct}} = - \frac{1}{2} \int_{z=\epsilon} \frac{d^4 k}{(2\pi)^4} \sqrt{\gamma} [2D^2 + \frac{D^2}{\ln(\epsilon\Lambda)} - \frac{1}{2} \ln(\epsilon\Lambda) F_{ij} F^{ij} + 2 \ln(\epsilon\Lambda) \bar{\lambda} \gamma^i k_i \lambda] . \quad (26)$$

The counterterms for the scalar components contribute also to the finite part of the boundary action so we finally get the result

$$S_{\text{ren}} = \frac{N^2}{8\pi^2} \int \frac{d^4 k}{(2\pi)^4} [d_0 \tilde{d}_0 - 2a_0^i \tilde{a}_{i2} - \frac{1}{2} a_0^i k^2 a_{i0} + \xi_0 \tilde{\chi}_1 + \bar{\chi}_1 \bar{\xi}_0] , \quad (27)$$

where we restored the normalization of the action which was neglected so far. The coefficient is due to the identification $\frac{1}{8\pi G_5} = \frac{N^2}{4\pi^2}$ which we will explain when treating the pure AdS example.

Twice differentiating with respect to the sources we finally get for the two-point functions

$$\langle J(k) J(-k) \rangle = \frac{N^2}{8\pi^2} \left(-2 \frac{\delta \tilde{d}_0}{\delta d_0} \right) , \quad (28)$$

$$\langle j_i(k) j_j(-k) \rangle = \frac{N^2}{8\pi^2} \left(2 \frac{\delta \tilde{a}_{i2}}{\delta a_0^j} + 2 \frac{\delta \tilde{a}_{j2}}{\delta a_0^i} + k^2 \eta_{ij} \right) , \quad (29)$$

$$\langle j_\alpha(k) \bar{j}_{\dot{\alpha}}(-k) \rangle = \frac{N^2}{8\pi^2} \left(\frac{\delta \tilde{\chi}_{1\alpha}}{\delta \xi_0^{\dot{\alpha}}} + \frac{\delta \bar{\chi}_{1\dot{\alpha}}}{\delta \xi_0^\alpha} \right) , \quad (30)$$

$$\langle j_\alpha(k) j_\beta(-k) \rangle = \frac{N^2}{8\pi^2} \left(\frac{\delta \tilde{\chi}_{1\alpha}}{\delta \xi_0^\beta} - \frac{\delta \tilde{\chi}_{1\beta}}{\delta \xi_0^\alpha} \right) . \quad (31)$$

An important comment about the fermionic correlators is in order. From the structure of the spinor equation of motion one can notice that the subleading mode $\tilde{\chi}_1$, which is determined by the full bulk equation, will always have a non-trivial dependence on the leading mode of opposite chirality $\bar{\xi}_0$, ensuring a non-zero value for the function $C_{1/2}$. On the other hand, already at this very general stage, we see that the only way to obtain a

non-zero $B_{1/2}$ is to have the mode $\tilde{\chi}_1$ to depend also holomorphically on the source ξ_0 . In the next section we will show that this is closely related to the presence of Majorana-like couplings in the $5d$ action that can arise only in the presence of a non-trivial profile for R-charged scalars. This should be expected, since a non-zero $B_{1/2}$ requires R-symmetry to be broken. We will see under which conditions non-trivial Majorana-like couplings of the bulk fermions can be produced.

3 Embedding the hidden sector in a gauged supergravity model

In this section we will introduce our model for the $5d$ bulk theory. As already noticed, the $5d$ gravity theory, besides the graviton multiplet, must contain at least one $\mathcal{N} = 2$ vector multiplet, which is dual to the multiplet of the conserved current of the boundary theory. As a necessary condition for the theory to be a consistent truncation of $10d$ type IIB supergravity, the matter content must include also one hypermultiplet which is the $\mathcal{N} = 2$ multiplet of the $10d$ dilaton, usually called universal hypermultiplet. In fact, enlarging the matter content to include an hypermultiplet is also necessary to the aim of finding interesting backgrounds, as we will see. Therefore, the minimal $5d$ content one should consider consists of $\mathcal{N} = 2$ supergravity coupled to a vector multiplet and a hypermultiplet.

In order to make our program concrete we consider a class of gauged supergravity theories studied in [23] which actually contains the minimal field content described above.⁵ We now briefly outline the main ingredients that specify our Lagrangian, whose form is dictated by the scalar manifold and the gauging. For further details we refer to [23, 24]. The scalars describe a non-linear sigma model with target space

$$\mathcal{M} = O(1, 1) \times \frac{SU(2, 1)}{U(2)}. \quad (32)$$

The manifold is a direct product of a very special manifold $\mathcal{S} = O(1, 1)$ and a quaternionic manifold $\mathcal{Q} = \frac{SU(2, 1)}{U(2)}$ spanned by the so-called universal hypermultiplet, which contains

⁵This class of theories has the virtue that, for some choices of the gauging, the resulting theory is believed to be a consistent truncation of the maximally gauged $\mathcal{N} = 8$ supergravity in $5d$ (and therefore of $10d$ type IIB compactified on a sphere).

the axio-dilaton. The \mathcal{S} factor is parametrized by the vector multiplet real scalar D with metric

$$ds_1^2 = dD^2, \quad (33)$$

whereas \mathcal{Q} is parametrized by the four real hyperscalars $q^X = (\phi, C_0, \eta, \alpha)$ with metric

$$ds_2^2 = g_{XY} dq^X dq^Y = \frac{1}{2} \cosh^2(\eta) d\phi^2 + \frac{1}{2} (2 \sinh^2(\eta) d\alpha + e^\phi \cosh^2(\eta) dC_0)^2 + 2d\eta^2. \quad (34)$$

where $\eta \geq 0$ and $\alpha \in [0, 2\pi]$. The scalar η is sometimes called squashing mode, since within 10d compactifications it is related to a squashing parameter of the internal compactification manifold. The isometries of this scalar manifold have a $U(2)$ maximal compact subgroup acting on \mathcal{Q} . Since the theory contains two vectors, one in the gravity multiplet and the other one in the vector multiplet, the maximal subgroup we can gauge is a $U(1) \times U(1)$. As a minimal set up we choose to gauge just the $U(1)$ corresponding to the shift symmetry

$$\alpha \rightarrow \alpha + c \quad (35)$$

of the above metric, which is a compact isometry because the scalar α is a phase.

The vector field of this $U(1)$ which acts non trivially on the scalar manifold is the graviphoton in the gravity multiplet, so that this gauge symmetry is dual to the R-symmetry of the boundary theory. On the other hand, in our simplified setting the $U(1)$ gauged by the vector belonging to the vector multiplet acts trivially on all supergravity fields (this does not mean that the vector multiplet is free, as we will see when we write its Lagrangian). Notice that the axio-dilaton is neutral under both $U(1)$'s while the complex scalar $\eta e^{i\alpha}$ is charged under the symmetry gauged in the bulk by the graviphoton. Therefore, a background with a non-trivial profile for the dilaton preserves the R-symmetry, while a non-trivial profile for η breaks it. For later reference let us notice that while the axio-dilaton is massless, and holographically dual to the hidden sector $\text{Tr} F_{ij}^2$ operator, the squashing mode η has $m^2 = -3$ and it is dual to the hidden sector gaugino bilinear. Hence, the leading mode for this field at the boundary would provide an explicit mass to the hidden gauginos (hence an explicit R-symmetry breaking term), while a sub-leading term would correspond to a VEV for the gaugino bilinear (hence a spontaneous R-symmetry breaking term).

Starting from our 5d Lagrangian, as already outlined in the previous section, there are basically two steps one should perform:

- First, we should find a non-supersymmetric background configuration with just the metric and some of the hyperscalars turned on. In order to do this we will truncate the Lagrangian to the relevant field content (provided this is consistent with the full set of equations) and extract the equations of motion which the background must satisfy.
- Second, we need to extract the linearized differential equations for the vector multiplet fluctuating on the background that we will find. To this aim, we will perform a different truncation of the Lagrangian setting all fields but the vector multiplet to their background values, and retain only the couplings which are no more than quadratic in the vector multiplet fields.

We will now present the explicit form of these truncated Lagrangians.

3.1 Lagrangian for the background

Let us start by setting to zero the whole vector multiplet, as well as the gravitino, the graviphoton and the fermions of the hypermultiplet. The phase α can be gauge-fixed to zero. The resulting truncated (Euclidean) action reads

$$S_{\text{b.g.}} = \int d^5x \sqrt{G} \left[-\frac{1}{2}R + L_{\text{kin}} + \mathcal{V} \right] \quad (36)$$

where the kinetic term is given in term of the metric (34) by $L_{\text{kin}} = \frac{1}{2}g_{XY}\partial_\mu q^X\partial^\mu q^Y$, that is

$$L_{\text{kin}} = \frac{1}{4} \left[4\partial_\mu\eta\partial^\mu\eta + \cosh^2(\eta)\partial_\mu\phi\partial^\mu\phi + e^{2\phi}\cosh^4(\eta)\partial_\mu C_0\partial^\mu C_0 \right]. \quad (37)$$

As a consequence of the gauging we have a non-trivial potential given by

$$\mathcal{V} = \frac{3}{4} (\cosh^2(2\eta) - 4\cosh(2\eta) - 5). \quad (38)$$

We end up with the following system of differential equations

$$R_{\mu\nu} = \frac{2}{3}\mathcal{V}G_{\mu\nu} + 2 \left(\partial_\mu\eta\partial_\nu\eta + \frac{1}{4}\cosh^2(\eta)\partial_\mu\phi\partial_\nu\phi \right), \quad (39)$$

$$\square\eta = \frac{1}{2}\frac{\partial\mathcal{V}}{\partial\eta} + \frac{1}{8}\sinh(2\eta)\partial_\mu\phi\partial^\mu\phi, \quad (40)$$

$$\square\phi = -2\tanh(\eta)\partial_\mu\eta\partial^\mu\phi, \quad (41)$$

where \square is the usual Klein-Gordon operator on a curved space

$$\square = \frac{1}{\sqrt{G}} \partial_\mu (\sqrt{G} G^{\mu\nu} \partial_\nu) . \quad (42)$$

The condition of asymptotically AdS background can be phrased by taking a metric of the form

$$ds_5^2 = \frac{1}{z^2} (dz^2 + F(z)(dx^i)^2) \quad (43)$$

with $F(z)$ approaching 1 at the boundary $z \rightarrow 0$. Therefore the solution to the above equations determine the three unknown functions ϕ , η and F of the radial coordinate z .

In the case of unbroken R-symmetry, $\eta = 0$, the above system of equations reduces exactly to the one considered in [10], and admits both a supersymmetric AdS solution with constant dilaton, as well as a singular dilaton domain-wall solution [10, 11]. The latter breaks both conformal invariance and (all) supersymmetry. Another interesting background is one where also the charged scalar η has a non-trivial profile. We will consider all these examples in turn. Notice that it is only in the R-breaking background that we need to fix the particular truncation and the form of the gauging which specify our 5d model. This is necessary to derive the potential for η and find its profile, and also to find out how it interacts with the fluctuations in the vector multiplet (at least at quadratic level in the vector superfield).

3.2 Quadratic Lagrangian for the vector multiplet

We now turn to the action describing the coupling of vector multiplet fluctuations to the background. To this end we fix F , ϕ and η to their (z -dependent) background value into the full Lagrangian, and retain only those terms involving the vector multiplet up to second order. The resulting (Euclidean) action can be divided in two pieces

$$S_{\text{quad}} = \int d^5x \sqrt{G} [L_{\text{min}} + L_{\text{int}}] . \quad (44)$$

The first one contains kinetic terms and mass terms for the fluctuations, and it is uniquely fixed by the dimensions of the dual operators and their minimal coupling to the metric to be L_{min} , eq. (13). The second one contains interactions with the scalars ϕ and η and takes the form

$$\begin{aligned} L_{\text{int}} = & \frac{1}{2} \delta M^2 D^2 - \delta m_D \bar{\lambda} \lambda \\ & - \frac{1}{2} (m_M \bar{\lambda} \lambda^c + v_M \bar{\lambda} (\not{\partial} \eta) \lambda^c + \tilde{v}_M \bar{\lambda} (\not{\partial} \phi) \lambda^c + c.c.) . \end{aligned} \quad (45)$$

where

$$\delta M^2 = 2(\cosh^2(2\eta) - \cosh(2\eta)) , \quad \delta m_D = -\frac{1}{2} \sinh^2(\eta) \quad (46)$$

$$m_M = i \sinh(\eta) , \quad v_M = -\frac{i}{\cosh(\eta)} , \quad \tilde{v}_M = \frac{i}{2} \sinh(\eta). \quad (47)$$

In the first line there are (z -dependent) shifts for scalar mass squared and Dirac fermion mass, whereas in the second line there are a Majorana mass term and additional Majorana-like couplings. We wrote the couplings in a 5d covariant manner, but one should bear in mind that η and ϕ are background values which actually can depend only on the radial coordinate, so that the additional terms are equivalent to 4d covariant terms constructed with a γ_5 matrix. Notice that all couplings (46)-(47) vanish if η is identically zero in the background.

From the action (44) we get the equations of motions

$$(\square + 4 - \delta M^2)D = 0 , \quad (48)$$

$$\frac{1}{\sqrt{G}} \partial_\mu (\sqrt{G} G^{\mu\rho} G^{\nu\sigma} F_{\rho\sigma}) = 0 , \quad (49)$$

$$(\not{D} - \frac{1}{2} - \delta m_D)\lambda - (m_M + v_M \not{\phi} \eta + \tilde{v}_M \not{\phi} \phi) \lambda^c = 0 , \quad (50)$$

where

$$\square = \frac{1}{\sqrt{G}} \partial_\nu (\sqrt{G} G^{\mu\nu} \partial_\mu) , \quad (51)$$

$$F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu , \quad (52)$$

$$\not{D} = e_a^\mu \gamma^a \left(\partial_\mu + \frac{1}{8} \omega_\mu^{cb} [\gamma_b, \gamma_c] \right) . \quad (53)$$

As already noticed, the 5d spinor is equal in form to a 4d Dirac spinor and it is often useful to rewrite its equation of motions in terms of chirality eigenstates, that is

$$\lambda = \begin{pmatrix} \chi \\ \bar{\xi} \end{pmatrix} , \quad \bar{\lambda} = - \begin{pmatrix} \xi & \bar{\chi} \end{pmatrix} , \quad \lambda^c = \begin{pmatrix} \xi \\ -\bar{\chi} \end{pmatrix} . \quad (54)$$

In terms of Weyl components χ and ξ , eq. (50) becomes

$$(z\partial_z - \frac{5}{2} + z\frac{F'}{F} - \delta m_D)\chi + i\frac{z}{\sqrt{F}}\sigma^i\partial_i\bar{\xi} - (m_M + v_M z\eta' + \tilde{v}_M z\phi')\xi = 0 , \quad (55)$$

$$(z\partial_z - \frac{3}{2} + z\frac{F'}{F} + \delta m_D)\bar{\xi} - i\frac{z}{\sqrt{F}}\bar{\sigma}^i\partial_i\chi - (m_M - v_M z\eta' - \tilde{v}_M z\phi')\bar{\chi} = 0 . \quad (56)$$

As can be seen from above equations, when Majorana-like couplings are turned on, not only $\bar{\xi}$ but also ξ appears in the equation for χ , and vice-versa. As we concluded in the previous section, we thus see that it is necessary to turn on a background for the scalar η in order to have correlators with a non-zero $B_{1/2}$.

3.3 Renormalized action with a non-trivial η

When the scalar η has a non-trivial profile and has non-vanishing leading boundary behavior, the renormalized action for the vector multiplet should be slightly modified.

The scalar η has $m^2 = -3$, and therefore its leading and subleading boundary behavior is

$$\eta \underset{z \rightarrow 0}{\simeq} \eta_0 z + \tilde{\eta}_2 z^3 + \dots \quad (57)$$

As the numerical analysis in the following sections will show, whenever the leading mode η_0 is present (a source term for the corresponding $\Delta = 3$ boundary operator, the hidden gaugino bilinear), the renormalized boundary action (27) should be modified by the following term

$$S_{\text{ren}}^\eta = \frac{N^2}{8\pi^2} \int \frac{d^4 k}{(2\pi)^4} [i\eta_0(\xi_0 \xi_0 - \bar{\xi}_0 \bar{\xi}_0)] . \quad (58)$$

Accordingly, the expression for the correlator (31) is modified to

$$\langle j_\alpha(k) j_\beta(-k) \rangle_\eta = \frac{N^2}{8\pi^2} \left(\frac{\delta \tilde{\chi}_{1\alpha}}{\delta \xi_{0\beta}} - \frac{\delta \tilde{\chi}_{1\beta}}{\delta \xi_{0\alpha}} + 2i\epsilon_{\alpha\beta} \eta_0 \right) . \quad (59)$$

The corrected expression (59) is necessary to ensure that the fermionic correlator properly goes to zero at large momenta, as dictated by supersymmetry restoration at high energy. The ultra-local term (58) can be seen as a counterterm which we add to the boundary action in order to reabsorb an unwanted contact term in the correlator. This counterterm only depends on quantities that are held fixed in the variational principle.

Notice that if the η profile has a leading boundary behavior proportional to $\tilde{\eta}_2$, which is holographically dual to a purely dynamical generation of an R-symmetry breaking VEV, no modification in the renormalized boundary action occurs. Still, having η a non trivial profile, $\tilde{\chi}_1$ would depend on ξ_0 , and hence the correlator (31) would be in general different from zero.

The origin of the additional term (58) can alternatively be motivated as follows. The

interaction Lagrangian (45) at linear order in η reads

$$L_{\text{int}}^{\text{lin}} = \frac{1}{2} \left(-i\eta\bar{\lambda}\lambda^c + i\bar{\lambda}(\not{\partial}\eta)\lambda^c - \frac{i}{2}\eta\bar{\lambda}(\not{\partial}\phi)\lambda^c + c.c. \right), \quad (60)$$

where we can actually neglect the third term, since in a background with a non-trivial dilaton profile, which necessarily behaves as $z\partial_z\phi = \mathcal{O}(z^4)$, this cannot contribute to the boundary action.

The key observation is that the following boundary term

$$S_{\text{reg}}^\eta = \int_{z=\epsilon} \frac{d^4k}{(2\pi)^4} \frac{i}{2} \epsilon^{-4} [\eta(\xi\xi - \bar{\xi}\bar{\xi} - \chi\chi + \bar{\chi}\bar{\chi})]_{z=\epsilon}. \quad (61)$$

is obtained if one integrates by parts the second term in (60). We note that this boundary term is now Majorana-like, in contrast with the usual one, eq. (25), which is Dirac-like. The term bilinear in χ is always vanishing at the boundary, but we notice that when $\eta \sim \epsilon$ the term bilinear in ξ is actually finite, and is exactly the term (58) after we restore the proper normalization.

One can easily verify that the action for η and λ with the interactions given in (60) vanishes on-shell up to quartic terms in those fields. Therefore, to this order of approximation, supplementing the renormalized boundary action by the counterterm (58) is equivalent to considering an interaction Lagrangian modified with respect to (60) by replacing the derivative interaction with the one obtained after integration by parts.

4 Holographic correlators in AdS

As a warm up exercise we want to compute the GGM two-point functions for a pure AdS background, which is a solution of eqs.(39)-(41) with $\phi = \eta = 0$. This exercise has several motivations. First of all it will enable us to verify that our machinery correctly reproduces what we expect from a conformal and supersymmetric case, namely eqs. (6). Second, the values for the correlators that we find in AdS will be the reference to confront with, when considering other backgrounds. In particular, each correlator will have to asymptote to those of the pure AdS case, at large momenta. Finally, the computations we perform in this section can be of interest in a different context, that is when conformality and supersymmetry breaking are implemented by a hard wall in AdS (for this perspective, see [25]).

The pure AdS solution is a trivial solution of our 5d effective model. However, in order to fix the overall normalization of correlators, it is useful to uplift it to the $AdS_5 \times S^5$ solution of 10d type IIB supergravity, which reads (see e.g. [10])

$$ds_{10}^2 = \frac{L^2}{z^2} (dz^2 + (dx^i)^2) + L^2 d\Omega_5^2 , \quad (62)$$

$$F_5 = \frac{N\sqrt{\pi}}{2\pi^3} (\text{vol}(S_5) + \frac{1}{z^5} d^4x \wedge dz) , \quad (63)$$

where the radius of AdS_5 is fix to be $L^4 = \frac{k_{10}N}{2\pi^{5/2}}$ by 10d Einstein equations. The overall constant in front of the 10d action is $1/2k_{10}^2$ so that, substituting the value of the 10d Newton constant in terms of the string theory parameters $k_{10} = \sqrt{8\pi G_{10}} = 8\pi^{7/2}g_s\alpha'^2$, we get $L^4 = 4\pi g_s N\alpha'^2$. Taking $L = \alpha' = 1$ we find $G_5 = \frac{\pi}{2N^2}$ and the overall constant in front of the 5d effective action is $1/8\pi G_5 = \frac{N^2}{4\pi^2}$.

In pure AdS the equations of motion (14), (15) and (20) are related to standard Bessel equations [26] and therefore it is possible to get analytic solutions for the fields

$$D(z, k) = -z^2 K_0(kz) d_0(k) ; \quad (64)$$

$$A_i(z, k) = zk K_1(kz) a_{0i}(k) ; \quad (65)$$

$$\bar{\xi}(z, k) = z^{5/2} k K_1(kz) \bar{\xi}_0(k) , \quad \chi = -z^{5/2} \sigma^i k_i K_0(kz) \bar{\xi}_0(k) . \quad (66)$$

The modified Bessel functions $K_\nu(x)$ can be written as power series which contain logarithmic modes for integer ν [26]. For our concerns, all we need to know is the behaviour of these functions near the origin

$$K_0(x) \underset{x \rightarrow 0}{\simeq} -\ln x + \ln 2 - \gamma + O(x^2) , \quad (67)$$

$$K_1(x) \underset{x \rightarrow 0}{\simeq} \frac{1}{x} \left[1 + \frac{x^2}{4} (2\ln x - 2\ln 2 + 2\gamma - 1) + O(x^4) \right] . \quad (68)$$

Using these expansions we get

$$\tilde{d}_0(k) = \left[-\frac{1}{2} \ln \left(\frac{\Lambda^2}{k^2} \right) - \ln 2 + \gamma \right] d_0(k) , \quad (69)$$

$$\tilde{a}_{i2}(k) = \frac{k^2}{2} \left[-\frac{1}{2} \ln \left(\frac{\Lambda^2}{k^2} \right) - \ln 2 + \gamma - \frac{1}{2} \right] a_{i0}(k) , \quad (70)$$

$$\bar{\xi}_2(k) = \frac{k^2}{2} \left[-\frac{1}{2} \ln \left(\frac{\Lambda^2}{k^2} \right) - \ln 2 + \gamma - \frac{1}{2} \right] \bar{\xi}_0(k) , \quad (71)$$

$$\tilde{\chi}_1(k) = \left[-\frac{1}{2} \ln \left(\frac{\Lambda^2}{k^2} \right) - \ln 2 + \gamma \right] \sigma^i k_i \bar{\xi}_0(k) . \quad (72)$$

Substituting these expressions into eqs.(28)-(31), we get for the two-point functions

$$\langle J(k)J(-k) \rangle = \frac{N^2}{4\pi^2} \left[\frac{1}{2} \ln \left(\frac{\Lambda^2}{k^2} \right) + \ln 2 - \gamma \right] ; \quad (73)$$

$$\langle j_i(k)j_j(-k) \rangle = -\frac{N^2}{4\pi^2} \left(\eta_{ij} - \frac{k_i k_j}{k^2} \right) k^2 \left[\frac{1}{2} \ln \left(\frac{\Lambda^2}{k^2} \right) + \ln 2 - \gamma \right] ; \quad (74)$$

$$\langle j_\alpha(k)\bar{j}_{\dot{\alpha}}(-k) \rangle = \frac{N^2}{4\pi^2} \sigma^i k_i \left[\frac{1}{2} \ln \left(\frac{\Lambda^2}{k^2} \right) + \ln 2 - \gamma \right] \quad (75)$$

$$\langle j_\alpha(k)j_\beta(-k) \rangle = 0 . \quad (76)$$

Our results are in agreement with CFT computations [27, 28]. Note that we can always subtract the constant contribution $\ln 2 - \gamma$ to the two-point functions by means of finite counterterms which preserve the $\mathcal{N} = 2$ supersymmetry of the bulk action, so these terms are inessential and will be ignored in what follows.

As expected for a supersymmetric background we find that the relations (6) are satisfied, and thus that both gaugino (8) and sfermion masses (7) are identically zero. In a general superconformal theory the OPE of the conserved current satisfies some general constraints which were studied in general in [29] and applied to the GGM formalism in [30]. In particular, if the hidden sector is exactly superconformal as it is the case for $\mathcal{N} = 4$ SYM, only the unit operator in the OPE of $J(x)J(0)$ can have an expectation value, leading to

$$C_0(x) = C_{1/2}(x) = C_1(x) = \frac{\tau}{16\pi^4 x^4} \rightarrow C_0(k^2) = C_{1/2}(k^2) = C_1(k^2) = \frac{\tau}{16\pi^2} \ln \left(\frac{\Lambda^2}{k^2} \right) , \quad (77)$$

where the coefficient τ associated to the unit operator has been exactly determined from 't Hooft anomaly in [31] and gives the contribution of the CFT matter to the beta function associated to the gauge coupling constant of the $U(1)$ subgroup of $SO(6)$ that we are gauging. For the AdS_5 case we find $\tau = 2N^2$. We note here that such a large number would be in contrast with keeping the SSM gauge couplings perturbative before unification. We will not comment on this further, besides saying that we are really trying to extract from this holographic approach qualitative features of correlators in strongly coupled hidden sectors, that we assume are a good approximation even outside the large N limit.

5 Holographic correlators in a dilaton-domain wall

In this section we do a step further and apply our machinery to a supersymmetry breaking background, which is also a solution of our 5d supergravity Lagrangian. In this case we keep a trivial profile for the squashing mode, $\eta = 0$, but allow for a non-trivial dilaton profile. We will see how the IR behavior of the correlators will change drastically with respect to their conformal expressions found in the previous section.

The dilaton-domain wall is in fact a solution of the full 10d type IIB supergravity found in [10, 11]. This is a singular solution with a non-trivial background for the dilaton ϕ which preserves the full $SO(6)$ R-symmetry. Upon dimensional reduction on S^5 we get the following 5d background

$$ds_5^2 = ds_5^2 = \frac{1}{z^2}(dz^2 + \sqrt{1-z^8}(dx^i)^2) , \quad (78)$$

$$\phi(z) = \phi_\infty + \sqrt{6} \operatorname{arctanh}(z^4) . \quad (79)$$

The metric goes to AdS_5 at the boundary $z \rightarrow 0$ and presents a naked singularity in the deep interior of the bulk, which we have set to $z = 1$ by adjusting one of the constants of integration. At the singularity the dilaton diverges

$$\lim_{z \rightarrow 1} \phi(z) = \infty . \quad (80)$$

The presence of the naked singularity signals a breakdown of the supergravity approximation and therefore the holographic interpretation of this background as a well-defined field theory could be problematic. It appears that this particular singularity is physically acceptable according to the two criteria of [32] and [33]. Respectively, its scalar potential is bounded from above (it is exactly zero), and g_{tt} is monotonously decreasing towards the singularity. The reason this solution has had some bad reputation is due to the fact that it fails another criterium put forward in [32], namely that it has no generalization with a horizon.

A possible physical interpretation of this background was discussed in [10, 12]. Suffices here to say that it describes a vacuum of a theory which in the UV coincides with $\mathcal{N} = 4$ SYM, where however a non-trivial VEV for $\operatorname{tr} F_{ij}^2$ is turned on triggering confinement and SUSY breaking. In the following we will probe some of its features by the explicit computation of the GGM correlators. This background is interesting for our program because it breaks, besides conformality, all the supersymmetries (as one can see from the

supersymmetry transformation of the dilatino) and it preserves the $SO(6)$ symmetry, so that we can consider an $\mathcal{N} = 2$ vector multiplet gauging a $U(1) \subset SO(6)$.

The effective action at the linearized level for the $\mathcal{N} = 2$ vector multiplet in the dilaton-domain wall is of the form (13), and the resulting equations of motion will take the schematic form

$$(\square_{DW} - 4)D \equiv \left(z^2 \partial_z^2 - \left(\frac{3 + 5z^8}{1 - z^8} \right) z \partial_z + \frac{z^2 (\partial_i)^2}{\sqrt{1 - z^8}} - 4 \right) D = 0 , \quad (81)$$

$$(\text{Max})_{DW} A_i \equiv \left(z^2 \partial_z^2 - \left(\frac{1 + 3z^8}{1 - z^8} \right) z \partial_z + \frac{z^2 (\partial_i)^2}{\sqrt{1 - z^8}} \right) A_i = 0 , \quad (82)$$

$$(\not{D}_{DW} - \frac{1}{2})\lambda \equiv \left(z \gamma_z \partial_z - 2 \frac{1 + z^8}{1 - z^8} \gamma_z + \frac{z}{(1 - z^8)^{1/4}} \gamma^i \partial_i - \frac{1}{2} \right) \lambda = 0 . \quad (83)$$

We note that the AdS equations are modified by terms of $O(z^8)$ in a near boundary expansion.

The second order equations for the fluctuations of the supergravity fields can be solved once two boundary conditions are specified.⁶ One boundary condition will always determine the leading term at the boundary, fixing the overall normalization of the solution. The second condition should be a regularity condition in the bulk. In the case under consideration this means to fix the behavior near the singular point $z = 1$.

Expanding eqs. (81)–(83) to the leading order in $1 - z \equiv y \rightarrow 0$ we get

$$(y^2 \partial_y^2 + y \partial_y) D = 0 , \quad (84)$$

$$(y^2 \partial_y^2 + \frac{1}{2} y \partial_y) A_i = 0 , \quad (85)$$

$$(y^2 \partial_y^2 + \frac{5}{4} y \partial_y - \frac{1}{8}) \bar{\xi} = 0 . \quad (86)$$

whose solutions are given in terms of two undetermined coefficients α and β as

$$D \underset{y \rightarrow 0}{\simeq} \alpha_0 \ln y + \beta_0 , \quad (87)$$

$$A_i \underset{y \rightarrow 0}{\simeq} \alpha_{i1} + \beta_{i1} y^{1/2} , \quad (88)$$

$$\bar{\xi} \underset{y \rightarrow 0}{\simeq} \alpha_{1/2} y^{-1/2} + \beta_{1/2} y^{1/4} . \quad (89)$$

The differential equations are well posed if we require, for all of the three fields, that

⁶For the sake of the argument that follows, we can convert the two first order equations for the spinors χ and $\bar{\xi}$ into a single second order equation for $\bar{\xi}$.

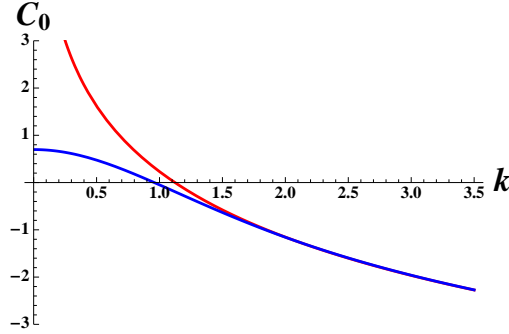


Figure 1: C_0 function: in red the AdS logarithm, in blue the dilaton domain wall result.

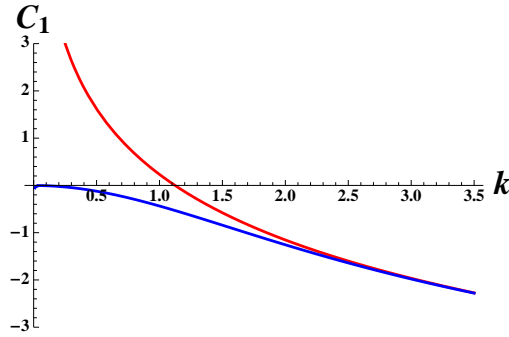


Figure 2: C_1 function: in red the AdS logarithm, in blue the dilaton domain wall result.

a linear combination of α and β vanishes.⁷ A condition giving a unequivocal choice for all of the three fields is requiring that both the field and its derivative are finite at the singularity. This condition can be satisfied for all of the three fields and their first derivatives, except for the first derivative of the fermion, which will diverge in any case. We thus select the choice of parameters $\alpha_0 = \beta_1 = \alpha_{1/2} = 0$.⁸

Once we specify the boundary conditions, a solution to eqs. (81)–(83) can be found numerically for any value of the parameter k corresponding to the $4d$ momentum. By using the holographic formulas (28)–(30) we can then plot the C_s functions.

We show the plots in figures 1, 2 and 3. In each graph we plot both the result for the supersymmetric AdS case, as well that for the dilaton domain wall solution. It is

⁷For instance $D = 0$ or $\partial\bar{\xi} = \text{const.}$ at the singularity are not suitable boundary conditions because they would kill both the coefficients.

⁸More general choices of the boundary conditions are in principle allowed (in bottom-up approaches for instance), and would give rise to different physics. For this perspective see [25].

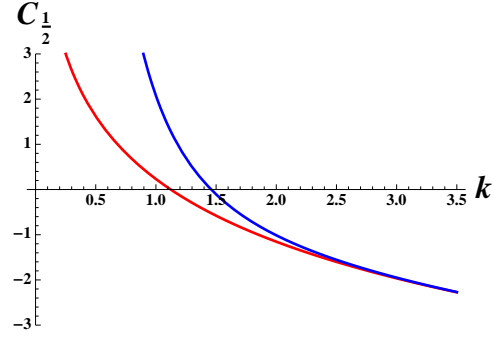


Figure 3: $C_{1/2}$ function: in red the AdS logarithm, in blue the dilaton domain wall result.

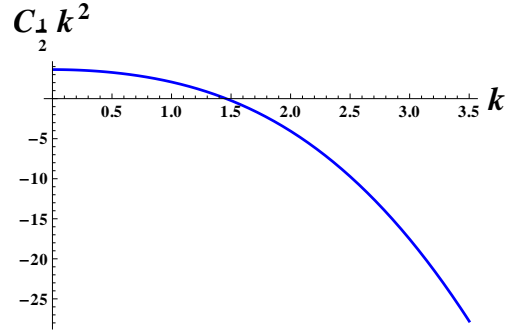


Figure 4: $k^2 C_{1/2}$: for $k \rightarrow 0$ it goes to a finite value, indicating that there is a $1/k^2$ pole in $C_{1/2}$ for the dilaton domain wall.

reassuring to note that the AdS tails for the three graphs correctly coincide and tend to their supersymmetric value.

One of the interesting results of the plots is the k^{-2} IR behavior of the fermionic correlator $C_{1/2}$. In figure 4 we plot $k^2 C_{1/2}$, which clearly shows this correlator has a $1/k^2$ pole at zero momentum. This kind of behavior is related to the existence of massless excitations carrying the same quantum numbers of the corresponding current. For the fermionic current j_α , this signals the existence of massless fermions, typically 't Hooft fermions, that compensate the global anomaly of the unbroken $U(1)_R$ -symmetry [13]. Note, in passing, that imposing the “wrong” boundary condition for the vector field fluctuations, namely $\alpha_1 = 0$, we would have gotten a k^{-2} pole also for C_1 . For the vector current this would be a massless Goldstone boson. This would imply the existence of Goldstone bosons associated to some broken global symmetry, which cannot be the case here since the original 10d background preserves the full $SO(6)$ (and hence our $U(1) \times U(1)_R$) symmetry.

While we cannot prove that there are indeed R-charged 't Hooft fermions in our strongly coupled theory, and just observe that the holographic analysis suggests them to be there, it is useful to refer to the full 10d background to get some more confidence about our result. From the 10d perspective there is a whole $SO(6)$ symmetry which the background preserves. Hence, at every scale there must exist massless fermions in the spectrum so to match the UV global anomaly. The UV fixed point is $\mathcal{N} = 4$ SYM, which has indeed a non-zero global anomaly for the $SO(6)$ current. At this point one may think that the $U(1)$ global symmetry of the hidden sector which we are eventually weakly gauging and identifying with the (simplified) SM gauge group is anomalous. This has not to be the case because having a non-zero $SO(6)^3$ anomaly still allows to consider a non-anomalous $U(1)$ subgroup inside $SO(6)$. On the contrary, our result suggests that (part of) the $SU(4)$ anomaly is transmitted to the $U(1)_R$ current. Let us emphasize that any other anomalous global symmetry would not provide a pole to the fermionic correlator $C_{1/2}$, which is neutral under any global symmetry but the R-symmetry. Hence, field theory expectations would suggest that when the R-symmetry is broken, R-charged 't Hooft fermions would not exist, and the pole in the fermionic correlator should vanish. We will come back to this point in the next section.

The Majorana gaugino mass, determined by $B_{1/2}$ through (8), vanishes because of unbroken R-symmetry. However, the pole in $C_{1/2}$ provides for a Dirac mass for the SSM

gaugino. This is very similar to any other model of R-symmetric Dirac gaugino masses, except that the massless fermion in the adjoint that must couple bilinearly with the gaugino is here a composite fermion generated at strong coupling. The phenomenological aspects of this spectrum will be discussed in more detail in [25]. Suffices to say, here, that the soft spectrum, in this situation, is very much reminiscent of that of gaugino mediation models. See [34, 14] for a discussion of Dirac gaugino masses in General Gauge Mediation.

Let us finally notice how different are the C_s in the dilaton domain wall background with respect to the ones in AdS, at large momentum. Numerically we find that

$$C_0 - 4C_{1/2} + 3C_1 \sim O(k^{-8}), \quad k \rightarrow \infty. \quad (90)$$

This is due to the fact that the correction of the domain wall metric with respect to the AdS one near the boundary is of $O(z^8)$. Note that since the dilaton does not enter the equations for the vector multiplet fluctuation, its $O(z^4)$ behaviour near the boundary does not influence the C_s . Another nice feature of the asymptotic behaviour (90) is that it makes the integral (7) nicely convergent in the UV.

Both the IR and the UV limit of the C_s functions could also be determined analytically by studying the equations (81)–(83) in the respective limits $k \rightarrow 0$ and $k \rightarrow \infty$. We delay this study to [25].

6 Holographic correlators in a dilaton/ η -domain wall

Let us discuss our last example, and look for a solution of eqs.(39)–(41) with a non-trivial profile for both the dilaton and the squashing mode. The latter breaks the R-symmetry so one should expect a very different behavior for the correlators.

In fact, in what follows we will only turn on a perturbative profile for the R-symmetry breaking scalar η , that is we consider only the linearized equation for η on the dilaton domain wall background, and neglect the backreaction of such a profile on the dilaton and the metric. As we are going to show, this will still be enough to provide a drastic change in the holographic correlators (nicely matching, again, field theory expectations).

The linearized equation for η is most conveniently written and solved using the following parametrization of the asymptotic AdS metric (with boundary at $r \rightarrow \infty$)

$$ds^2 = (dr^2 + e^{2r}(dx^i)^2) \quad (91)$$

and reads

$$\eta''(r) + 4\coth(4r)\eta'(r) + 3\eta(r) - \frac{3}{2(\sinh(4r))^2}\eta(r) = 0. \quad (92)$$

The solution depends on two integration constants A and B and is given by

$$\eta(r) = (e^{8r} - 1)^{\frac{1}{4}\sqrt{\frac{3}{2}}} \left[A {}_2F_1\left(\frac{2+\sqrt{6}}{8}, \frac{4+\sqrt{6}}{8}, \frac{3}{4}, e^{8r}\right) + B {}_2F_1\left(\frac{4+\sqrt{6}}{8}, \frac{6+\sqrt{6}}{8}, \frac{5}{4}, e^{8r}\right) \right], \quad (93)$$

where ${}_2F_1$ is the hypergeometric function.

Changing variables to the usual $z = e^{-r}$ radial coordinate, one can verify that indeed this solution has the expected behavior (57) near the boundary, with η_0 and $\tilde{\eta}_2$ expressed as linear combinations of A and B . On the other hand, studying the equation near the singularity $y = 1 - z \rightarrow 0$ one finds the following behavior

$$\eta \underset{y \rightarrow 0}{\simeq} \alpha y^{\frac{1}{4}\sqrt{\frac{3}{2}}} + \beta y^{-\frac{1}{4}\sqrt{\frac{3}{2}}}, \quad (94)$$

with α and β which are in turn linear combinations of A and B . If one imposes the boundary condition at the singularity so to meet the criterion on the boundedness of the potential [32], that is $\beta = 0$, one finds a relation between A and B which imposes both η_0 and $\tilde{\eta}_2$ to be turned on at the boundary (indicating that R-symmetry is broken explicitly in the hidden sector). This implies that in doing the holographic renormalization procedure one should bear in mind the discussion in section 3.3 and augment the boundary action by the term (58).

Plugging our results in the formulas for the holographic correlators (11) and (12), it is easy to see that C_0 and C_1 are unaffected. On the other hand, both fermionic correlators are modified. As shown in figure 5 the correlator $B_{1/2}$ has now a non-trivial dependence on the momenta. Consistently with expectations, it reaches a finite value at zero momentum (hence providing non-vanishing Majorana mass to SSM gauginos), and falls off to zero at $k \rightarrow \infty$. On the other hand, the pole at $k^2 = 0$ in $C_{1/2}$ has now disappeared (see figure 6). This is consistent with field theory intuition: R-symmetry being broken, 't Hooft fermions, if any, cannot couple to the j_α current and provide zero momentum poles in $C_{1/2}$. We see the fact that as soon as η has a non-trivial profile the correlators $B_{1/2}$ becomes non-vanishing and, *at the same time*, the pole in $C_{1/2}$ vanishes, as a remarkable and non-trivial agreement with expectations from the field theory side.

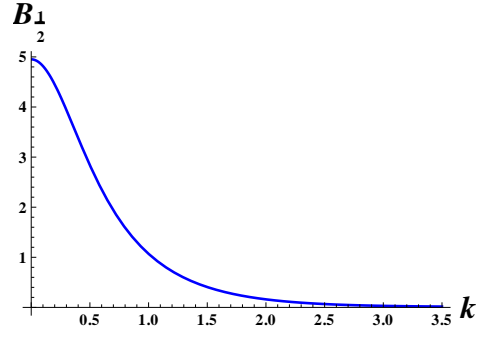


Figure 5: $B_{1/2}$ obtained on a background with a non-trivial profile for η .

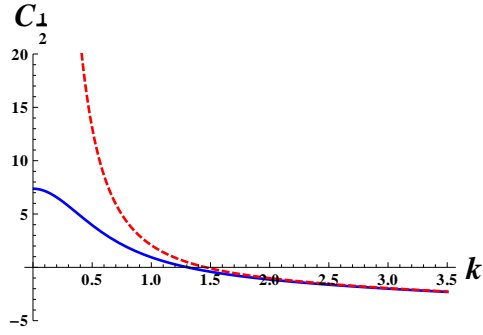


Figure 6: The red dashed line is $C_{1/2}$ in the background with $\eta = 0$, while the blue solid line is the result when a profile for η is turned-on.

7 Conclusions and perspectives

In this paper we have laid out the procedure for computing two-point functions of operators belonging to a conserved current supermultiplet in a strongly coupled field theory, using holography. This was aimed, primarily, at providing a holographic computation for correlators that play a role in models of (general) gauge mediation.

We have been working in the context of 5d consistent truncations of type IIB string theory and focused our attention on supersymmetry breaking asymptotically AdS backgrounds. We have found that when R-symmetry is unbroken, the SSM gauginos generically acquire a Dirac mass by coupling to composite fermions, which manifest themselves as massless poles in the fermionic correlator $C_{1/2}$. Sfermions have masses derived from an integral which converges very nicely in the UV, and are dominated by the pole of the fermionic correlator, providing a spectrum which is reminiscent of gaugino mediation models. On the contrary, for R-symmetry breaking backgrounds the pole in $C_{1/2}$ disappears, while the R-breaking correlator $B_{1/2}$ acquires a non-trivial profile, hence providing Majorana mass to SSM gauginos. All our results are in remarkable agreement with field theory expectations [13, 14].

These techniques can be applied, in principle, to several other models and can also find applications in different contexts. Possible further research directions are as follows.

In the present work, we have adopted a top-down approach, and worked in the context of consistent truncations of type IIB 10d string theory. A different approach is to use a bottom-up set up. For instance, one could use a model of dynamical SUSY breaking reminiscent of AdS/QCD set ups. The simplest one is an AdS background with a IR hard wall⁹ (HW). In [25] we perform an analysis similar to the one we performed in the present work for hard wall backgrounds. The benefit in considering HW setups is that on the one hand the correlation functions can be established analytically; on the other hand, at the price of loosing any clear string embedding, there is much more flexibility (for instance, one can allow a profile for η providing a VEV for the hidden gauginos bilinear and not a source term).

A natural next step is to try and generalize the present holographic approach to backgrounds which are not asymptotically AdS. Indeed, a superconformal theory cannot

⁹GGM realizations in warped geometries have already been considered in [35]. The essential difference between [35] and our approach is that in the former, the SUSY-breaking sector is realized as a field theory in the warped geometry, while in our scenario it is defined by the geometry itself.

break supersymmetry spontaneously (because of the tracelessness of the stress-energy tensor). Thus, the cases considered here and in [25] are nice toy-models but cannot be the full story for a genuine SUSY breaking hidden sector. It would be interesting, in this respect, to extend our analysis to cascading backgrounds, as those considered in [36, 37].

Eventually, the addition of probe or backreacting D7 branes to represent the SSM gauge groups will also be a necessary ingredient, especially if one wants to make contact with the original set up of holographic gauge mediation [36, 37]. Indeed, in 5d backgrounds descending from string theory without explicit D-brane sources, the maximal global symmetry one can have is $SO(6) \simeq SU(4)$, i.e. the rank is not big enough to match that of the SSM gauge group. Adding D7 branes is thus necessary in this top-down approach, even though it makes the mediation of SUSY breaking less direct and neat.

Finally, let us emphasize that the GGM framework can be seen as an instance of situations in which fields in a visible sector are coupled linearly to composite operators in a strongly coupled hidden sector, and observables are extracted from correlators of such composite operators. This kind of setting is found also in other scenarios, like technicolor-like theories and composite Higgs models, so that one can envisage broader BSM applications along the lines of the holographic calculation we performed here.

Acknowledgements

We would like to thank Francesco Bigazzi, Aldo Cotrone, Stefano Cremonesi, Gianguido Dall’Agata, Nick Evans, Anton Faedo, Alberto Mariotti, Carlos Nunez, Gary Shiu and Brian Wecht for useful discussions. We would also like to thank Kostas Skenderis and Marika Taylor for correspondence on [9]. The research of R.A. and D.R. is supported in part by IISN-Belgium (conventions 4.4511.06, 4.4505.86 and 4.4514.08) and by a “Mandat d’Impulsion Scientifique” of the F.R.S.-FNRS. R.A. is a Research Associate of the Fonds de la Recherche Scientifique–F.N.R.S. (Belgium). M.B., L.D.P. and F.P. acknowledge partial financial support by the MIUR-PRIN contract 2009-KHZKRX and by the ESF network “Holograv”.

References

- [1] J. M. Maldacena, *The Large N limit of superconformal field theories and supergravity*, Adv. Theor. Math. Phys. **2** (1998) 231 [Int. J. Theor. Phys. **38** (1999) 1113] [hep-th/9711200].
- [2] E. Witten, *Anti-de Sitter space and holography*, Adv. Theor. Math. Phys. **2** (1998) 253 [hep-th/9802150].
- [3] S. S. Gubser, I. R. Klebanov and A. M. Polyakov, *Gauge theory correlators from noncritical string theory*, Phys. Lett. B **428** (1998) 105 [hep-th/9802109].
- [4] M. Bianchi, D. Z. Freedman and K. Skenderis, *How to go with an RG flow*, JHEP **0108** (2001) 041 [hep-th/0105276].
- [5] M. Bianchi, D. Z. Freedman and K. Skenderis, *Holographic renormalization*, Nucl. Phys. B **631** (2002) 159 [hep-th/0112119].
- [6] K. Skenderis, *Lecture notes on holographic renormalization*, Class. Quant. Grav. **19** (2002) 5849 [hep-th/0209067].
- [7] P. Meade, N. Seiberg, D. Shih, *General Gauge Mediation*, Prog. Theor. Phys. Suppl. **177** (2009) 143-158. [arXiv:0801.3278 [hep-ph]].
- [8] P. McGuirk, *Hidden-sector current-current correlators in holographic gauge mediation*, Phys. Rev. D **85** (2012) 045025 [arXiv:1110.5075 [hep-th]].
- [9] K. Skenderis and M. Taylor, *Holographic realization of gauge mediated supersymmetry breaking*, to appear.
(see also <http://hep.physics.uoc.gr/mideast5/talks/Saturday/Taylor.pdf>)
- [10] S. S. Gubser, *Dilaton driven confinement*, hep-th/9902155.
- [11] A. Kehagias and K. Sfetsos, *On Running couplings in gauge theories from type IIB supergravity*, Phys. Lett. B **454** (1999) 270 [hep-th/9902125].
- [12] N. R. Constable and R. C. Myers, *Exotic scalar states in the AdS / CFT correspondence*, JHEP **9911** (1999) 020 [hep-th/9905081].

- [13] M. Buican and Z. Komargodski, *Soft Terms from Broken Symmetries*, JHEP **1002** (2010) 005 [arXiv:0909.4824 [hep-ph]].
- [14] K. Intriligator and M. Sudano, *General Gauge Mediation with Gauge Messengers*, JHEP **1006** (2010) 047 [arXiv:1001.5443 [hep-ph]].
- [15] K. Skenderis and M. Taylor, *Kaluza-Klein holography*, JHEP **0605** (2006) 057 [hep-th/0603016].
- [16] H. J. Kim, L. J. Romans and P. van Nieuwenhuizen, *The Mass Spectrum of Chiral $N=2$ $D=10$ Supergravity on S^{**5}* , Phys. Rev. D **32** (1985) 389.
- [17] M. Gunaydin and N. Marcus, *The Spectrum of the s^{**5} Compactification of the Chiral $N=2$, $D=10$ Supergravity and the Unitary Supermultiplets of $U(2, 2/4)$* , Class. Quant. Grav. **2** (1985) L11.
- [18] E. Shuster, *Killing spinors and supersymmetry on AdS* , Nucl. Phys. B **554** (1999) 198 [hep-th/9902129].
- [19] P. Breitenlohner and D. Z. Freedman, *Positive Energy in anti-De Sitter Backgrounds and Gauged Extended Supergravity*, Phys. Lett. B **115** (1982) 197.
- [20] P. Breitenlohner and D. Z. Freedman, *Stability in Gauged Extended Supergravity*, Annals Phys. **144** (1982) 249.
- [21] M. Henningson and K. Sfetsos, *Spinors and the AdS / CFT correspondence*, Phys. Lett. B **431** (1998) 63 [hep-th/9803251].
- [22] M. Henneaux, *Boundary terms in the AdS / CFT correspondence for spinor fields*, In *Tbilisi 1998, Mathematical methods in modern theoretical physics* 161-170 [hep-th/9902137].
- [23] A. Ceresole, G. Dall'Agata, R. Kallosh and A. Van Proeyen, *Hypermultiplets, domain walls and supersymmetric attractors*, Phys. Rev. D **64** (2001) 104006 [hep-th/0104056].
- [24] A. Ceresole and G. Dall'Agata, *General matter coupled $N=2$, $D = 5$ gauged supergravity*, Nucl. Phys. B **585** (2000) 143 [hep-th/0004111].

- [25] R. Argurio, M. Bertolini, L. Di Pietro, F. Porri and D. Redigolo, to appear.
- [26] G. N. Watson, *A Treatise on the Theory of Bessel Functions*, Cambridge University Press, 1922.
- [27] W. Mueck and K. S. Viswanathan, *Conformal field theory correlators from classical scalar field theory on $AdS(d+1)$* , Phys. Rev. D **58** (1998) 041901 [hep-th/9804035].
- [28] W. Mueck and K. S. Viswanathan, *Conformal field theory correlators from classical field theory on anti-de Sitter space. 2. Vector and spinor fields*, Phys. Rev. D **58** (1998) 106006 [hep-th/9805145].
- [29] J. -F. Fortin, K. Intriligator and A. Stergiou, *Current OPEs in Superconformal Theories*, JHEP **1109** (2011) 071 [arXiv:1107.1721 [hep-th]].
- [30] J. -F. Fortin, K. Intriligator and A. Stergiou, *Superconformally Covariant OPE and General Gauge Mediation*, JHEP **1112** (2011) 064 [arXiv:1109.4940 [hep-th]].
- [31] D. Anselmi, J. Erlich, D. Z. Freedman and A. A. Johansen, *Positivity constraints on anomalies in supersymmetric gauge theories*, Phys. Rev. D **57** (1998) 7570 [hep-th/9711035].
- [32] S. S. Gubser, *Curvature singularities: The Good, the bad, and the naked*, Adv. Theor. Math. Phys. **4** (2000) 679 [hep-th/0002160].
- [33] J. M. Maldacena and C. Nunez, *Supergravity description of field theories on curved manifolds and a no go theorem*, Int. J. Mod. Phys. A **16** (2001) 822 [hep-th/0007018].
- [34] K. Benakli and M. D. Goodsell, *Dirac Gauginos in General Gauge Mediation*, Nucl. Phys. B **816** (2009) 185 [arXiv:0811.4409 [hep-ph]].
- [35] M. McGarrie and D. C. Thompson, *Warped General Gauge Mediation*, Phys. Rev. D **82** (2010) 125034 [arXiv:1009.4696 [hep-th]].
- [36] F. Benini, A. Dymarsky, S. Franco, S. Kachru, D. Simic and H. Verlinde, *Holographic Gauge Mediation*, JHEP **0912** (2009) 031 [arXiv:0903.0619 [hep-th]].
- [37] P. McGuirk, G. Shiu and Y. Sumitomo, *Holographic gauge mediation via strongly coupled messengers*, Phys. Rev. D **81** (2010) 026005 [arXiv:0911.0019 [hep-th]].